ATTACK-RESILIENT OBSERVER PRUNING FOR PATH-TRACKING CONTROL OF WHEELED MOBILE ROBOT

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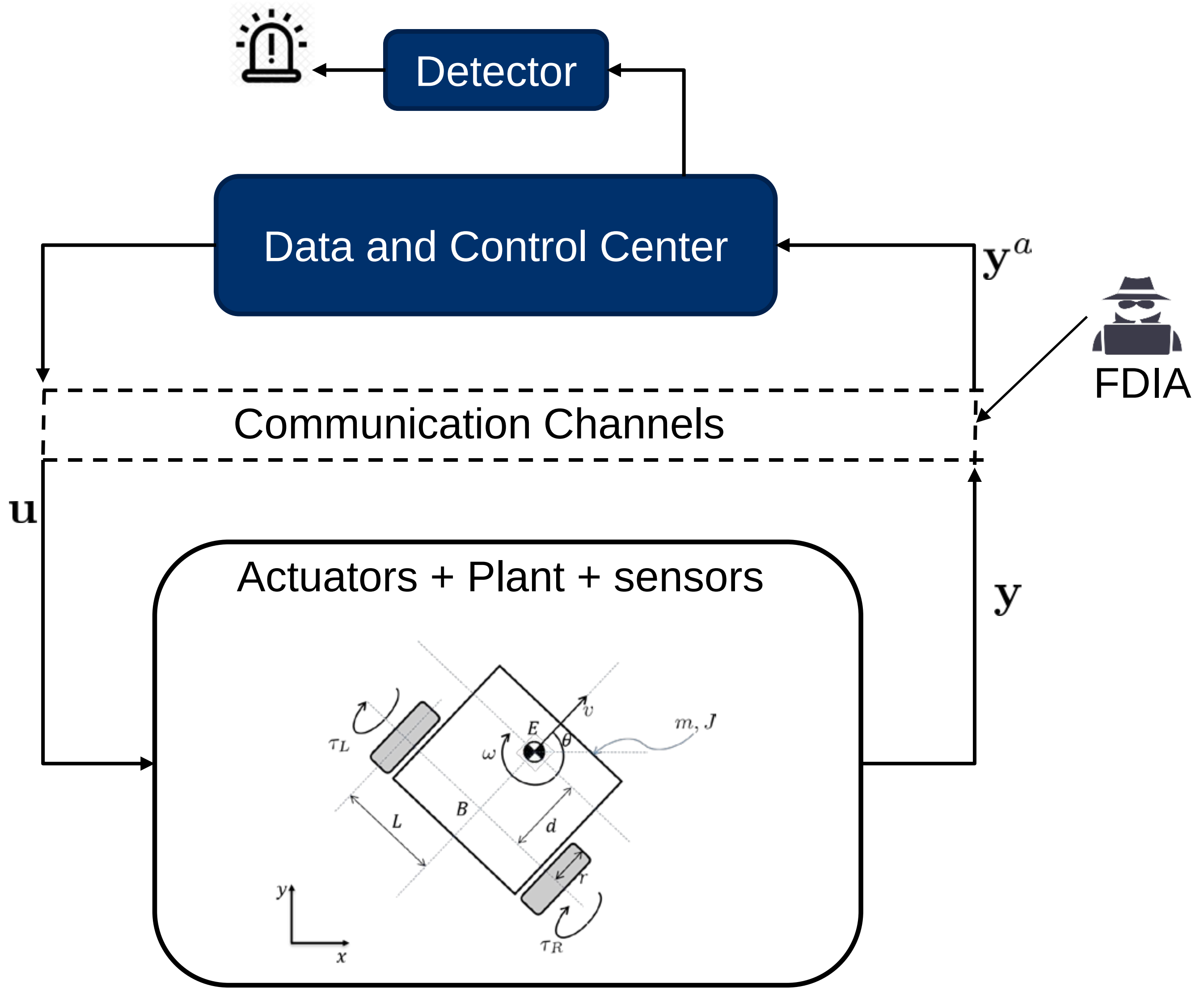
Motivation

Output feedback control for WMR based on communication network



System and problem statement

- System construction
  - Plant model
  - Path-tracking control design
  - Measurement model
  - Observer scheme
- False data injection attack
  - Residual-based monitor
  - FDIA design
- Resilient estimation
  - Compressed sensing
  - Attack detection and localization

Plant model: (dynamic and kinematic model)

$$M\dot{\mathbf{q}} + D(\mathbf{q})\mathbf{q} = B\tau$$

$$\begin{bmatrix} \dot{\theta} \\ \cdots \\ \dot{\mathbf{z}} \end{bmatrix} = \bar{C}(\theta)\mathbf{q} = \begin{bmatrix} 0 \quad 1 \\ \cdots \\ C(\theta) \end{bmatrix} \mathbf{q} \qquad (1)$$

$$\mathbf{q} = \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$M = \begin{bmatrix} m & 0 \\ 0 & md^2 + J \end{bmatrix}, \quad D = \begin{bmatrix} 0 & -md\omega \\ md\omega & 0 \end{bmatrix}$$

$$B = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix}, \quad C(\theta) = \begin{bmatrix} \cos(\theta) & -d\sin(\theta) \\ \sin(\theta) & d\cos(\theta) \end{bmatrix}.$$

Error System: (dynamic and kinematic model)

Target trajectory: $[\theta_d \quad x_d \quad y_d]^\top$

$$\mathbf{e} = \begin{bmatrix} \theta - \theta_d \\ \mathbf{z} - \mathbf{z}_d \end{bmatrix} = \begin{bmatrix} \mathbf{e}_\theta \\ \mathbf{e}_\mathbf{z} \end{bmatrix} \qquad (2)$$

$$\widetilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$$

Path-tracking control

Path-tracking control design:

$$\tau = B^{-1}(M\mathbf{u} + D\mathbf{q}) \qquad (3)$$
$$\mathbf{u} = -k_q(\mathbf{q} - \mathbf{q}_d) + \dot{\mathbf{q}}_d - \bar{C}(\theta)^\top \mathbf{e}$$

$$\mathbf{q}_d = C^{-1}(\theta)(\dot{\mathbf{z}}_d - k_e\mathbf{e_z})$$
$$\dot{\mathbf{q}}_d = -k_e(\dot{C}^{-1}(\theta)\mathbf{e_z} + \mathbf{q}) + C^{-1}(\theta)[\ddot{\mathbf{z}}_d + (k_e + C(\theta)\dot{C}^{-1}(\theta))\dot{\mathbf{z}}_d]$$

Stability criterion: Consider the control law given in (3), if the control gains k_q and k_e are chosen as $k_q > 0, k_e > 0$, then the tracking errors in (2) converges to zero asymptotically. Furthermore, the generalized velocities tracking error $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$ converges to zero asymptotically with $\dot{\mathbf{z}}_d = C(\theta)\mathbf{q}_d$ satisfied in the limit.

Proof: Candidate Lyapunov function: $V = \frac{1}{2}\|\tilde{\mathbf{q}}\|^2 + \frac{1}{2}\|\mathbf{e}\|^2$

$$\dot{V} \leq -k_q\|\tilde{\mathbf{q}}\|^2 - k_e\|\mathbf{e}_z\|^2 \qquad (5)$$

(Negative semi-definite)

$\tilde{\mathbf{q}}, \mathbf{e} \in \mathcal{L}_\infty$

$$\left.\begin{aligned} \dot{\tilde{\mathbf{q}}} &= -k_q\tilde{\mathbf{q}} - \bar{C}(\theta)^\top\mathbf{e} \in \mathcal{L}_\infty \\ \dot{\mathbf{e}} &= \bar{C}(\theta)\tilde{\mathbf{q}} - k_e\begin{bmatrix}0\\ \mathbf{e_z}\end{bmatrix} \in \mathcal{L}_\infty \end{aligned}\right\}$$

$\mathbf{e}$ and $\tilde{\mathbf{q}}$ are uniformly continuous.

Barbalat's Lemma → $\mathbf{e}(t) \to 0, \tilde{\mathbf{q}}(t) \to 0.$

$$V - V(0) \leq -\int_0^t \left(k_q\|\tilde{\mathbf{q}}(\tau)\|^2 + k_e\|\mathbf{e_z}(\tau)\|^2\right)d\tau \text{ implies } \tilde{\mathbf{q}}, \mathbf{e_z} \in \mathcal{L}_2.$$

Measurement system and monitor

Measurement model:

$$\mathbf{y} := f(\mathbf{x}) + \mathbf{v} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1/4r & L/4r \\ 1/4r & -L/4r \\ \cos(\theta) & -d\sin(\theta) \\ \sin(\theta) & d\cos(\theta) \end{bmatrix} \cdot \mathbf{q} + \mathbf{v}$$

Observer: Unscented Kalman Filter

$$\min E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^\top]$$

Residual-based monitor:

$$\Psi_T : \{Y_T, U_T\} \mapsto \{\Psi_1, \Psi_2\}$$

$Y_T \in R^{m \times T}$: measurements during time horizon T

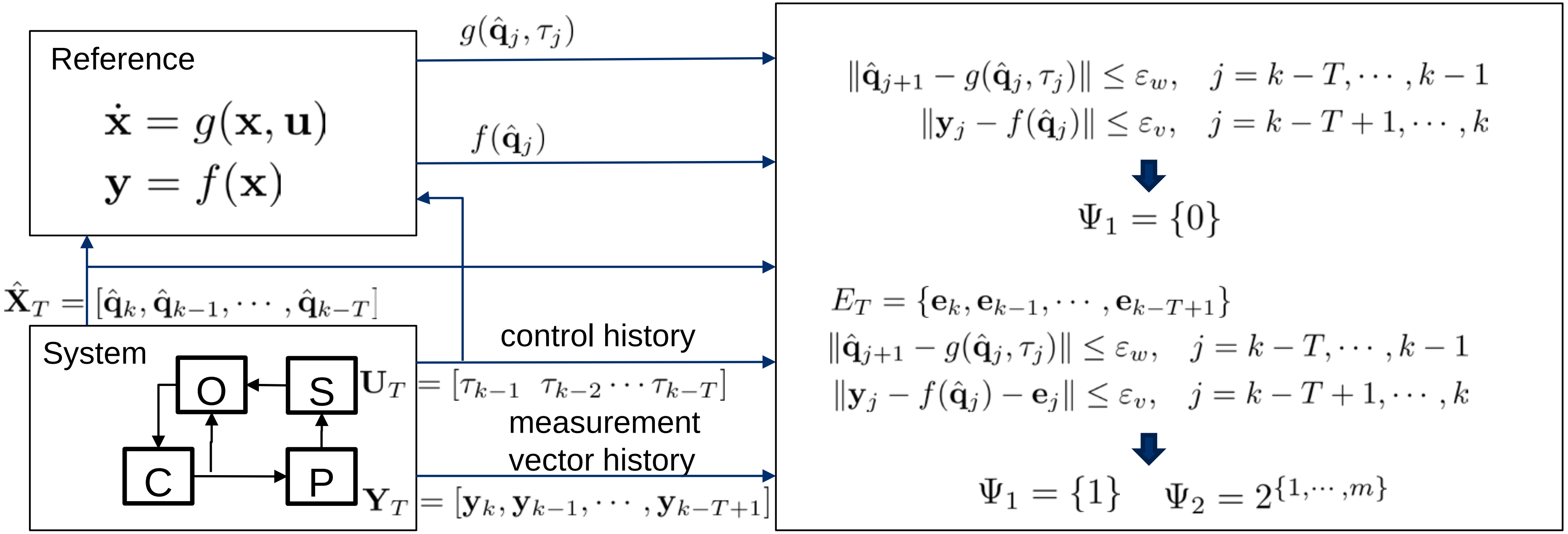
$U_T \in R^{l \times T}$: controlled inputs during time horizon T

$\Psi_1 = \{0(safe), 1(unsafe)\}$ Alarm

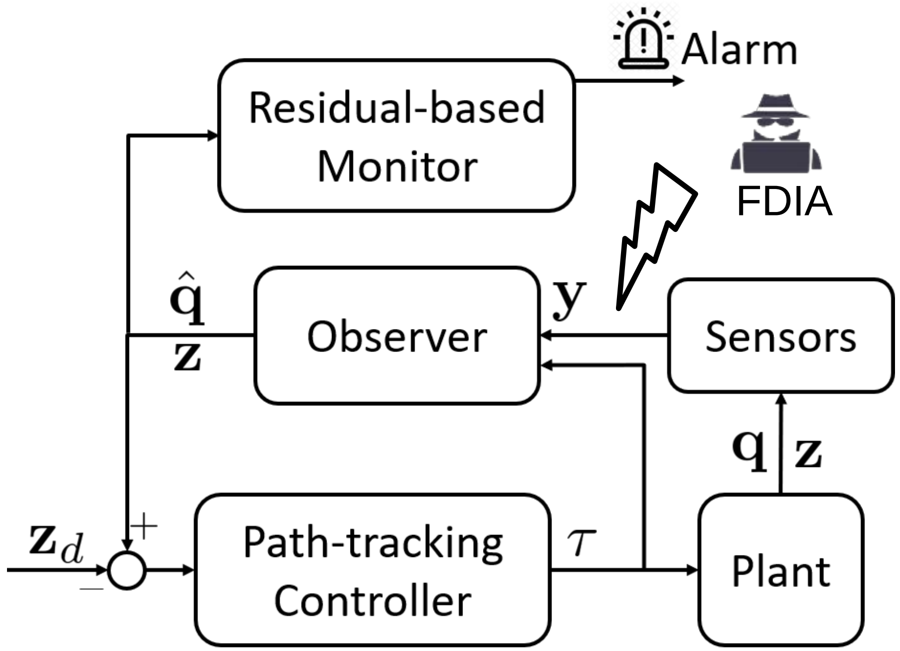
$\Psi_2 = 2^{\{1,2,\cdots,m\}}$ Alarm location

Monitor

Residual-based monitor:



False-data Injection Attacks



FDIA Design:

$$Y_f = H\mathbf{x}_k + G\mathbf{u}_f + e$$

$$H = \begin{bmatrix} C_d \\ C_d A_m \\ C_d A_m^2 \\ \vdots \\ C_d A_m^{T_f} \end{bmatrix}, G = T_s \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_d B_m & 0 & \cdots & 0 \\ C_d A_m B_m & C_d B_m & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_d A_m^{T_f-1} B_m & C_d A_m^{T_f-2} B_m & \cdots & C_d B_m \end{bmatrix}$$

$$H = [U_1 \;\; U_2] \begin{bmatrix} \Sigma_1 \\ 0 \end{bmatrix} V$$

Given selction vector $\mathcal{T}$ under upper-bound of attack percentage P_A, one successful FDIA can be constructed by:

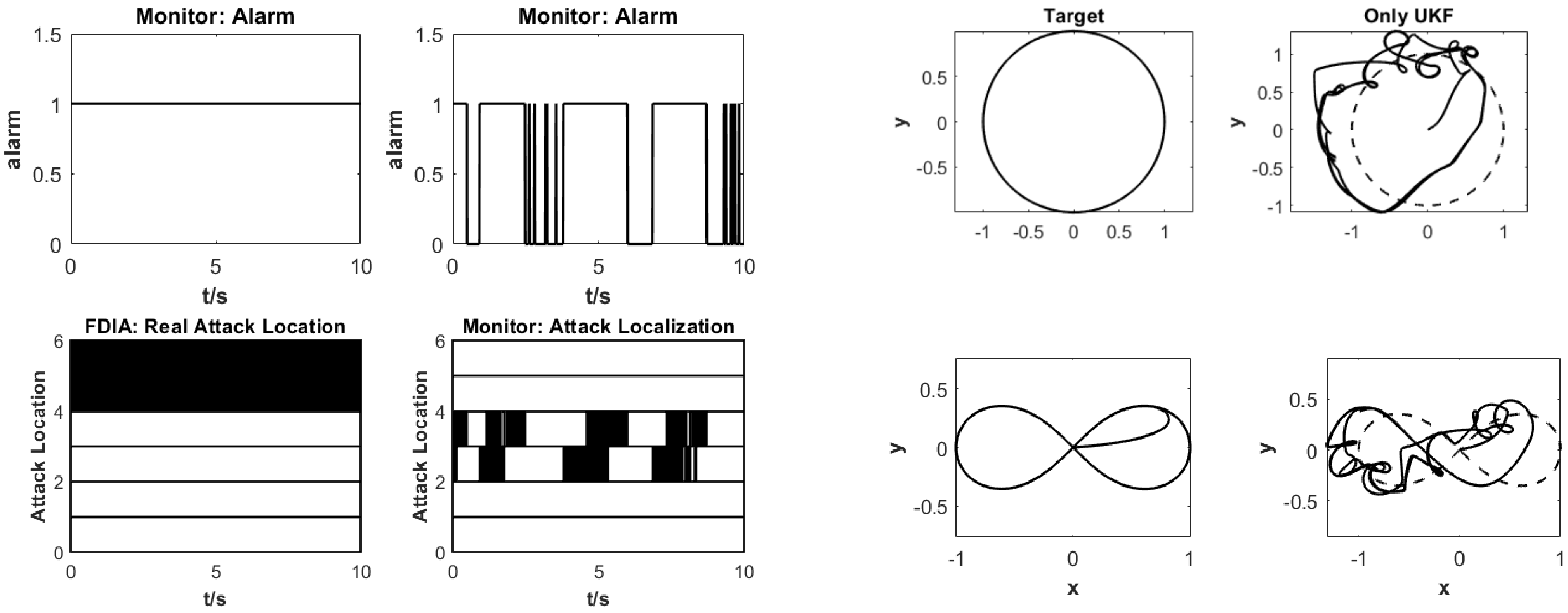
$$\text{Maxmize}: \quad \|U_{1,\mathcal{T}.}^{\top}\mathbf{e}\|,$$

$$\text{Subject to}: \quad \|(U_{2,\mathcal{T}.}^{\top})_j \mathbf{e}_j\| \leq \tau_j, \quad j \in \mathcal{T}$$

(where, τ_j is the escaping parameter of bad data detector defined by $\|(U_{2,\mathcal{T}.}^{\top})_j\| \cdot \varepsilon_v$)

False-data Injection Attacks

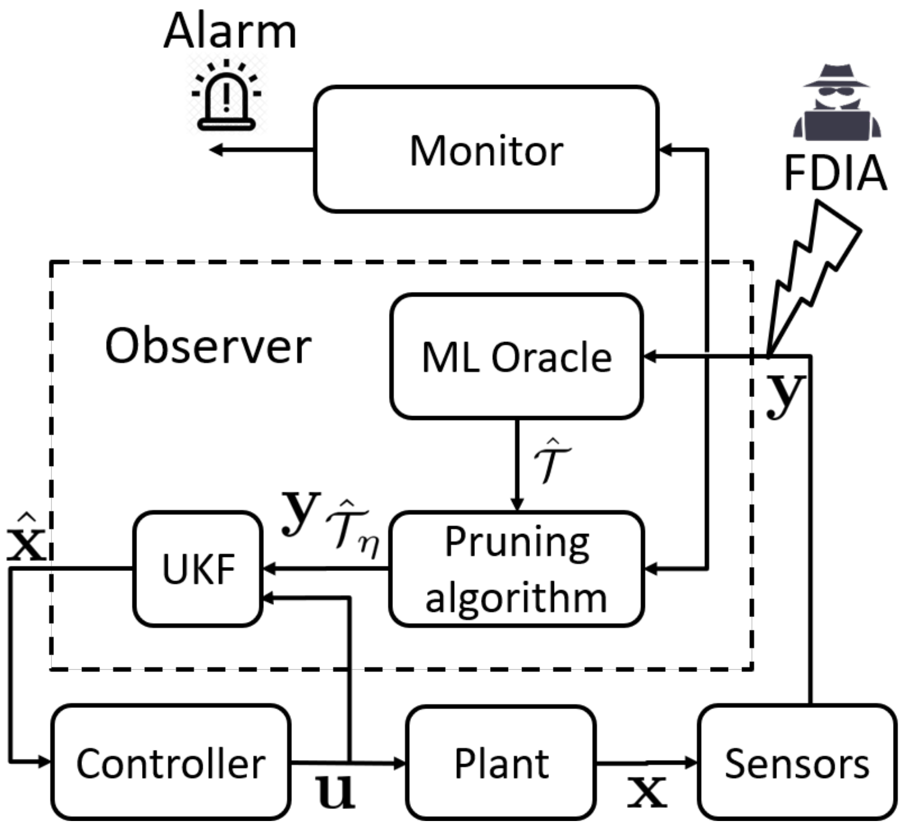
FDIA Design:



Resilient estimation

Resilient estimation：

Attack localization ✚ Robust estimation algorithm



Main results

Uncertainty of Oracle：

Actual support

$\mathcal{T}^c$

$$\mathbf{q}_i = \begin{cases} 1 & \text{if } \ i \in \mathcal{T}^c \\ 0 & \text{otherwise} \end{cases}$$

Oracle support

$\hat{\mathcal{T}}^c$

$\hat{\mathbf{q}}$

Uncertainty model

$$\mathbf{q}_i = \epsilon_i \hat{\mathbf{q}}_i + (1 - \epsilon_i)(1 - \hat{\mathbf{q}}_i)$$

$\epsilon_i \sim \mathcal{B}(1, p_i)$ is the agreement defined by: $\epsilon_i = \begin{cases} 1 & \text{if } \ \hat{\mathbf{q}}_i = \mathbf{q}_i \\ 0 & \text{if } \ \hat{\mathbf{q}}_i = 1 - \mathbf{q}_i \end{cases}$

probability mass function: $Pr\left(\sum_{i=1}^{m} \epsilon_i = k - 1\right) = \mathbf{r}(k), k = 1, \cdots, m+1$

$$\mathbf{r} = \prod_{i=1}^{m} P_i \cdot \mathbf{g}_1 * \cdots * \mathbf{g}_i * \cdots * \mathbf{g}_m, \ \mathbf{r} \in R^{m+1}, \text{ and } \mathbf{g}_i = \begin{bmatrix} \frac{1-P_i}{P_i} \\ 1 \end{bmatrix}$$

Main results

Pruning Algorithm：

1. Obtaining reliable trust parameter：

Given reliability level $\eta \in (0,1)$, return the maximum integer $l_\eta \leq N$ such that l_η safe nodes are correctly localized with a probability of at least η:

$$l_\eta = \max\left\{k \mid \Pr\left\{\sum_{i\in\hat{\mathcal{T}}^c} \epsilon_i \geq k\right\} \geq \eta\right\}$$
$$= \max\left\{k \mid \sum_{i=1}^{k+1} \mathbf{r}_{\hat{\mathcal{T}}^c}(i) \leq 1-\eta\right\}$$

2. Pruning：

A new support is obtained through a robust extraction:

$$\hat{\mathcal{T}}_\eta^c = \left\{\text{argsort} \downarrow (\mathbf{p} \circ \hat{\mathbf{q}})\right\}_1^{l_\eta}$$

where, $\{\cdot\}_1^{l_\eta}$ is an index extraction from the first elements to l_η elements.

Remark： If the underlying machine learning algorithm works better than random flip of fair coin, then through pruning algorithm, it follows

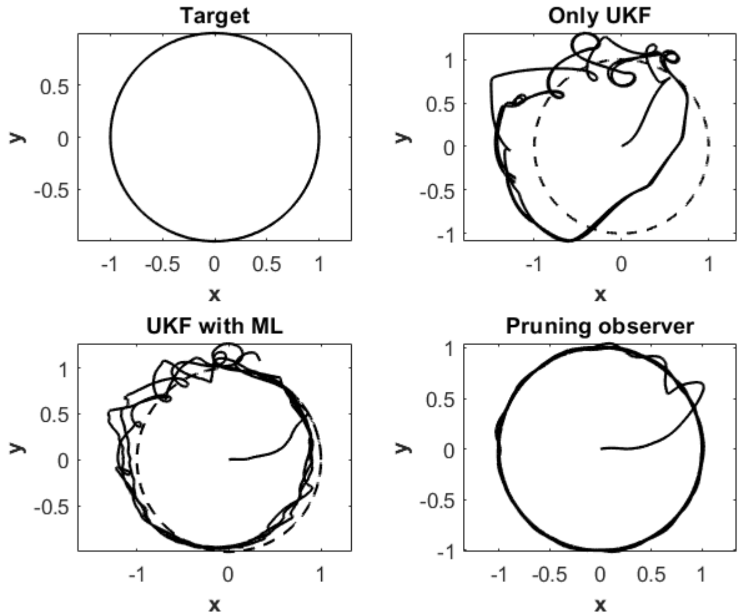
$$\Pr\{\hat{\mathcal{T}}_\eta^c \cap \mathcal{T} = \emptyset\} \geq \eta.$$

Simulation

Circle path tracking:

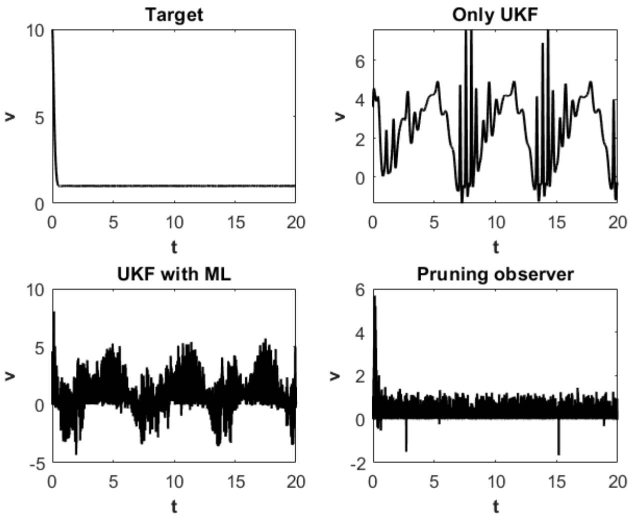
$$\text{Target: } \begin{bmatrix} t \\ cos(t) \\ sin(t) \end{bmatrix}$$

path tracking

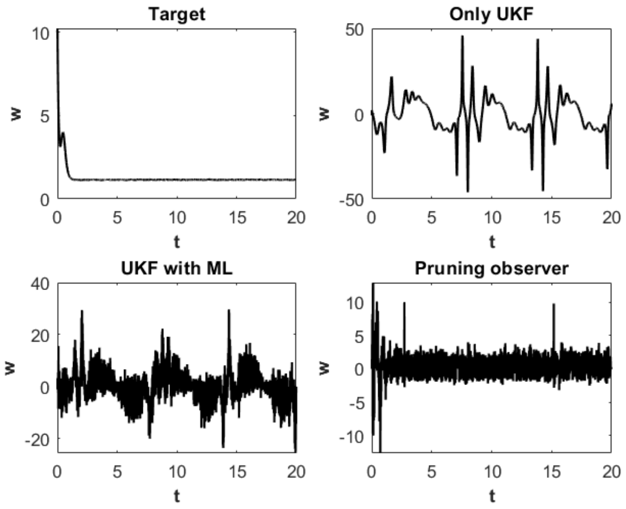


State estimation

Forward velocity (v)



angular velocity (w)



Conclusion

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  An attack-resilient control and estimation scheme for path-tacking task of WMR under false data injection attacks

  - Stable path-tracking control system for non-holonomic WMR
  - Optimization-based FDIA design scheme
  - Pruning-based observer design using UKF as the underlying observer

- Future work
  - **Measurement redundancy:** include L1-minimization with pruning algorithm
  - **Robustness:** L1-based Receding horizon estimation scheme
  - **Concurrency:** Concurrent learning model

THANK YOU

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Simulation codes: https://github.com/ZYblend/Resilient-Pruning-Observer-against-for-WMR-under-FDIA